Correction for the Sherlock-Monro Algorithm for Generating the Space of Real Orthonormal Wavelets

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Abstract

Evaluation of the Sherlock-Monro algorithm *orthogen* for generating the space of real orthonormal wavelets of length $N = 2J$ from J angle parameters revealed that it was valid only for $1 \leq J \leq 3$. A correction is provided and validated for all $J \geq 1$ with tests for orthonormality and perfect reconstruction. Some additional comments are offered discussing wavelet filter design algorithms based on angular parameterizations and those based on spectral factorizations.

1 Problem

Sherlock and Monro [1] presented an algorithm for generating the space of real orthonormal wavelets of length $N = 2J$ from J angular parameters $\alpha_j \in [0, 2\pi)$. More precisely, their algorithm generates the FIR filter coefficients for the lowpass scalets corresponding to highpass wavelets with one vanishing moment only if $\sum_{j=0}^{J-1} \alpha_j = \pi/4$. Contrary to the angular parameterization of Zou and Tewfik [2, 3], the Sherlock-Monro method is not valid for wavelets with higher moments.

To clarify and demonstrate their MATLAB algorithm *orthogen* [1, p.1718], they provided a flow diagram for the case $J = 4$, and 2-D contour plots for the case $J = 3$. Higher order examples $J \geq 5$ were not tested. Independent tests of orthonormality were not performed. When the author's suite of tests [4] were applied to the filters output by the Sherlock-Monro algorithm, all cases $J \geq 4$ failed all relevant tests.

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2 Solution

Examination of the program listing for *orthogen* and comparison with the flow diagram revealed the source of the problem. The algorithm should be corrected by the insertion of the statement *butterflybase = butterflybase+2;* as the last statement inside the *for butterfly* loop. Thus, the corrected loop sequence should read:

```
for butterfly = 1:nbutterflies,
   hlo = h(butterflybase);
   hhi = h(butterflybase+1);
   h(butterflybase)   = c*hhi - s*hlo;
   h(butterflybase+1) = s*hhi + c*hlo;
   butterflybase = butterflybase+2;
end,
```

With this correction, all cases $1 \leq J \leq 100$ passed orthonormality and perfect reconstruction tests for the filter coefficients output when angles $\alpha_j \in [0, 2\pi)$, uniformly distributed and $(\pi/4)$-sum normalized, were input to the corrected *orthogen*. Each output filter was tested for M-shift orthogonality error $\mathrm{moe}(\cdot)$, M-band reconstruction error $\mathrm{mre}(\cdot)$, and vanishing moments numbers $\mathrm{vmn}(\cdot)$ in an $N \times M$ filter bank $\mathbf{F}$ where $M = 2$ for these CQF wavelet filter banks. For $\mathbf{F}$ of size 200×2 for the case $J = 100$, $\mathrm{moe}(\mathbf{F}) = 4.4 \times 10^{-16}$, $\mathrm{mre}(\mathbf{F}) = 4.4 \times 10^{-16}$, and $\mathrm{vmn}(\mathbf{F}) = [0, 1]$ were observed confirming the expected orthonormality and perfect reconstruction for $\mathbf{F}$ with one vanishing moment on the wavelet $\mathbf{f}_1$ and none on the scalet $\mathbf{f}_0$.

3 Comments

Unfortunately, these filter characteristics do not guarantee that the filter will have other desireable characteristics such as maximal frequency-domain selectivity or minimal time-frequency uncertainty. Although the parameter-space constraint on the angles for one vanishing moment on the wavelet may insure some time-domain regularity and other desireable characteristics with relevance to low order filters with small N, it does not necessarily for high order filters with large N. Searching a parameter space for large J becomes increasingly computationally expensive. Thus, finding a filter with desireable characteristics becomes more difficult because of the unrestricted search space. Although the angular parameterization of Zou and Tewfik [2] does impose constraints for more than one vanishing moment, they did not present any filter examples for $J > 2$.

In contrast, Daubechies wavelets with a wide variety and combination of desireable filter characteristics can be readily computed via spectral factorization of a symmetric positive polynomial [5, 6]. Despite the criticism of other authors [2, 1] regarding the numerical instabilities inherent in spectral factorization, so far the method remains more useful in generating higher order wavelets with more than one vanishing moment. Clearly, each of the different algorithms has advantages and disadvantages. Thus, the most prudent and practical position to adopt would be that of verifying for each algorithm its utility in terms of the class of filters and range of filter lengths N for which the algorithm is valid, the possible combinations of desired filter characteristics for which a search can be done, and the computational complexity of the search for filters with those characteristics. This task has been completed for the Daubechies wavelets computed via spectral factorization [7].

References

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