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Randomized Signal Classes for Evaluating the Performance of Wavelet Shrinkage Denoising Methods

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Abstract

Previous simulation experiments for the comparison of wavelet shrinkage denoising methods have used *fixed* signal classes defined by adding instances of noise to a single test signal. New simulation experiments are reported here with *randomized* signal classes defined by adding instances of noise to instances of randomized test signals. As expected, significantly greater variability in the performance of the denoising methods was observed. Statistically valid comparisons must be conducted with respect to this variability. Use of randomized, rather than fixed, signal classes should yield more realistic and meaningful results.*

Keywords: wavelet domain thresholding, shrinkage denoising, non-parametric signal estimation.

1 Introduction

Denoising by thresholding in the wavelet domain has been developed principally by Donoho *et al.* [1, 2, 3, 4]. In [1], they introduced *RiskShrink* with the minimax threshold, *VisuShrink* with the universal threshold, and discussed both hard and soft thresholds in a general context that included ideal denoising in both the wavelet and Fourier domains. In [2], they introduced *SureShrink* with the SURE threshold, *WaveJS* with the James-Stein threshold, and *LPJS* also with the James-Stein threshold but in the Fourier domain instead of the wavelet domain. The procedure *LPJS* was renamed *FourJS* (analogous to *WaveJS*) for consistency of mnemonics by Taswell [5], who also labelled the various denoising procedures respectively 'RIS', 'VIS', 'IWD', 'IFD', 'SUR', 'WJS', and 'FJS' for use as abbreviations.

The first Monte Carlo simulation experiment comparing any of these denoising procedures was performed by Taswell and published in the article by Donoho and Johnstone [1, Table 4, page 448; Acknowledgements, page 450]. Various other experiments have since been performed by other authors (see discussion and references in [4]). Most of this work has examined four test signals originally called 'Doppler', 'HeaviSine', 'Blocks', and 'Bumps' by Donoho and Johnstone [1]. The latter was renamed more descriptively as 'Spires' by Taswell [5]. All of the experiments on these test signals, including the most recent experiments [5], examined only these fixed test signals rather than defined classes of randomized test signals.

To address this deficiency in the design of the simulation experiments, new classes of randomized test signals are introduced here, and used in new experiments which provide a more appropriate evaluation of the performance of the denoising methods. For example, instead of using just one instance of Spires with the particular values of the peak height, width, and location parameters originally defined in [1], multiple instances of Random Spires are used in the experiments with randomized values of the peak height, width, and location parameters. The use of such randomized signal classes in the experiments results in a more realistic assessment of the variability of performance that can be expected for the different denoising methods.

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2 Methods

Randomized signal classes, called 'Random Blocks', 'Random Spires', and 'Random HeaviSine', were defined to generate signals analogous to the original 'Blocks', 'Spires', and 'HeaviSine'. Figure 1 displays the original nonrandomized versions in the top row of subplots, and one instance each of the randomized versions in the bottom row of subplots. Table 1 lists the mathematical formulae for the test signal classes. These formulae are valid for both the randomized and original nonrandomized versions with the appropriate choice of parameters. Table 2 lists the MATLAB pseudocode expressions for the set of parameters chosen for the randomized classes used in the experiments reported here. Signals from the signal classes were corrupted with additive Gaussian noise (SNR = 10), and then denoised with the various denoising methods. Performance of the denoising procedures on the signal classes was studied as a function of sample size $n = 2^J$ for $J = 8, \ldots, 14$. The signal-to-noise ratio (SNR) was used as the objective figure of merit for comparing the original signal $S[n]$ with the denoised estimate $\hat{S}[n]$, computed as

$$\mathrm{SNR} = 10 \log_{10} \frac{\sum_n |S[n]|^2}{\sum_n |S[n] - \hat{S}[n]|^2}$$

with results averaged over all signal instances and reported in decibels (dB) with means, standard deviations, and coefficients of variation. Experimental results reported here were computed with Version 4.6a1 of the WAVBOX Software Library [6] running in Version 5.3.0 of the MATLAB technical computing environment.

3 Results

Figure 2 presents results expressed as mean ±1 standard deviation of the SNR values for all trials. As expected, the coefficients of variation (ratios of standard deviation to mean) were significantly larger for the randomized signal classes when compared with the original fixed signal classes. Table 3 demonstrates that the increase was approximately 2 – 10 fold for VIS. Similar results were observed for the other methods. Such large differences necessarily impact the statistical validity of comparisons of the methods. For example, when comparing performance of the methods on the signal classes, the error bars (representing ± 1 standard deviation) do not overlap for Blocks but do overlap for Random Blocks, implying that any differences between the methods are not statistically significant for this randomized signal class under the experimental conditions investigated. However, the method VIS does perform significantly worse than the other methods for Random Spires. The relatively poor performance of VIS also affects the class Random Blocks albeit not to the same degree of significance.

4 Conclusion

New randomized signal classes have been introduced to facilitate the statistically valid comparison of the performance of wavelet shrinkage denoising methods. Use of a fixed signal class can result in misleading inferences from invalid comparisons. Careful attention should be focused on the use of appropriately defined signal classes when evaluating denoising methods in simulation experiments. These experiments can then be used to compare the performance of various denoising methods. When there is no statistically significant difference in the methods' performance on the defined signal class, other criteria such as computational complexity should be used to select a preferred method. Moreover, if a particular method can be demonstrated to perform significantly worse than other competing methods, such as shown here for VIS on Random Spires, it would be prudent to exclude it from further consideration for use as a denoising method for the signal class and experimental conditions investigated.

References

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Table 1: Mathematical Formulae for Signal Classes

Name	Function	Kernel	Parameters
Blocks	$f(t) = \sum_{m=1}^{M} h_m K(t - p_m)$	$K(s) = (\text{sgn}(s) + 1)/2$	M, h_m, p_m
Spires	$f(t) = \sum_{m=1}^{M} h_m K((t - p_m)/w_m)$	$K(s) = (\lvert s\rvert + 1)^{-4}$	M, h_m, p_m, w_m
HeaviSine	$f(t) = h_1 \sin(p_1 \pi t) + \sum_{m=2}^{M} h_m K(t - p_m)$	$K(s) = \text{sgn}(s)$	M, h_m, p_m

Table 2: Pseudocode Expressions for Parameters Used in Randomized Versions

Name	h_m	p_m	w_m	m
Blocks	5*sign(rand(1,11)-0.5).*rand(1,11)	sort(rand(1,11))		$1, \ldots, 11$
Spires	5*rand(1,11)	sort(rand(1,11))	0.05*rand(1,11)	$1, \ldots, 11$
HeaviSine	4	4		1
	2*sign(rand(1,2)-0.5).*rand(1,2)	sort(rand(1,2))		2,3

Table 3: VIS Denoising: Coefficients of Variation for SNR Values.

	Blocks		Spires		HeaviSine	
J	Fixed	Random	Fixed	Random	Fixed	Random
8	2.8501e-002	1.1718e-001	7.9789e-002	1.0950e-001	5.2748e-002	6.5265e-002
9	3.4988e-002	1.3192e-001	5.5483e-002	8.5289e-002	3.5886e-002	5.7628e-002
10	2.3885e-002	1.3916e-001	3.5810e-002	7.8460e-002	3.0110e-002	6.2710e-002
11	1.8842e-002	1.4474e-001	1.9456e-002	5.7327e-002	2.3419e-002	6.8886e-002
12	1.5847e-002	1.2818e-001	1.6622e-002	5.0351e-002	2.4020e-002	5.3679e-002
13	1.0527e-002	8.9286e-002	1.0516e-002	4.1403e-002	1.2165e-002	7.0534e-002
14	6.7975e-003	1.1607e-001	7.8823e-003	5.1193e-002	1.4964e-002	1.1089e-001

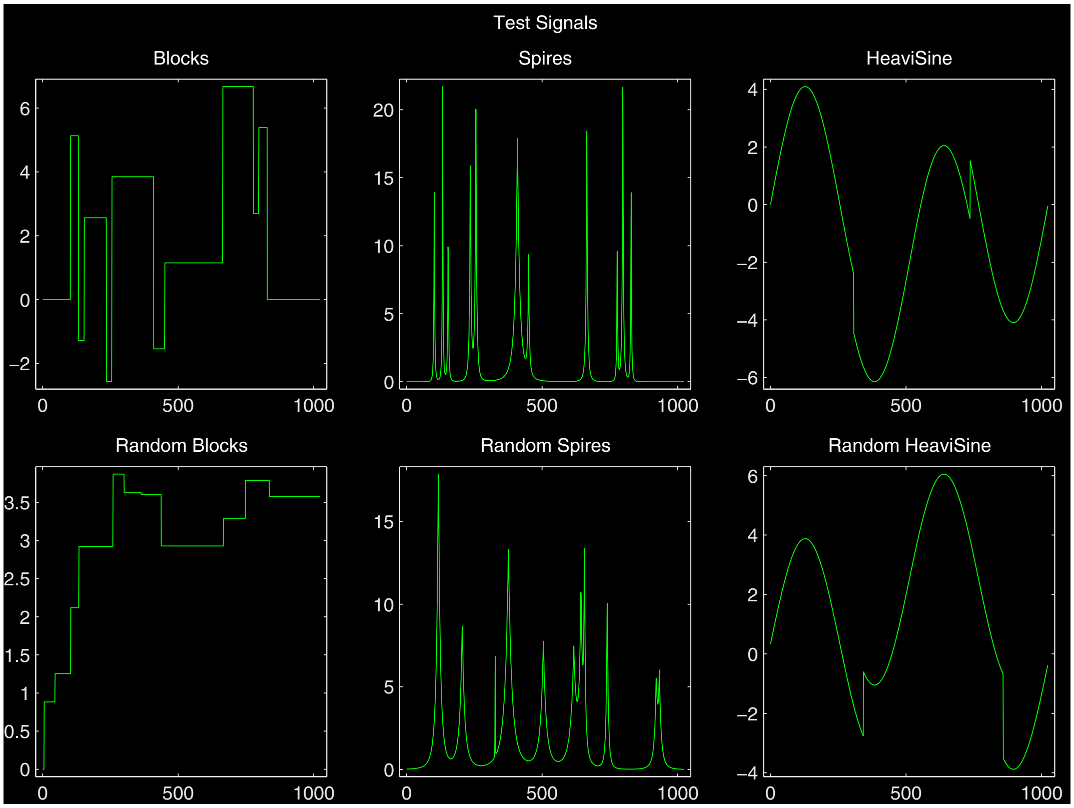

Figure 1: Standardized Test Signals.

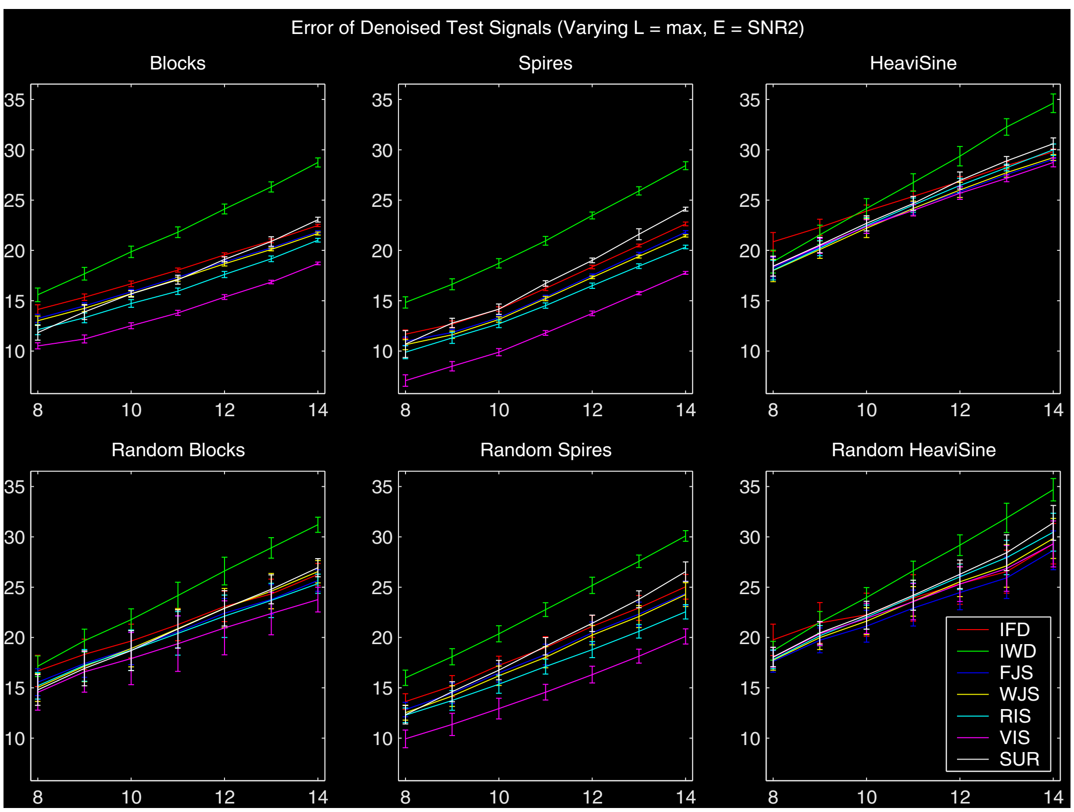

Figure 2: Mean SNR for Denoised Test Signals.